Monte Carlo Simulation Methods as an Estimation Tool for Capital Requirements

Petra Matušková
VŠB – TU Ostrava
Faculty of Economics, Finance department
Sokolská třída 33
Ostrava, 701 21
Czech Republic
e-mail: petra.matuskova@vsb.cz

Abstract
With the development of economy and European market it comes to an integration and harmonization in the area of financial markets, and at the same time the emphasis is put on credibility, transparency and stability of financial institutions. Risk management and solvency play a key role in the financial institutions and their functions. To ensure the solvency of financial institutions, and therefore the ability to meet their obligations at any time, the institutions must hold a certain amount of capital for risk coverage. Capital requirements are regulated by legislative framework and the main method for their determination is the Value at Risk. The Monte Carlo simulation is flexible and valuable tool for estimating Value at Risk. There are a few methods which improving estimation which is acquired by basic procedure for Monte Carlo simulation. The aim of paper is determination of capital requirements for currency risk in insurance and bank sector by various methods of Monte Carlo Simulation.

Keywords: Capital requirements, Value at Risk, Monte Carlo simulation
JEL codes: G17, G32

1. Introduction

With the development of economy and European market it comes to an integration and harmonization in the area of financial markets, and at the same time the emphasis is put on credibility, transparency and stability of financial institutions, which at the time of political and economic instability and not ending economic crises is getting bigger. In order to ensure protection of clients and all other aforementioned aspects, it is necessary to determine certain rules for entrepreneurship in this area, including designation of supervisory institutions, which monitor and control compliance of determined rules. Significant tool of regulation in insurance and banking sectors is monitoring of their solvency, thus insurance company's or a bank's ability to secure permanent fulfillment of liabilities by their own sources.

In banking and insurance sector the way of regulation and monitoring are gradually regulated at international level approximately from the 70's of last century. With the development of both markets and inception of new instruments, the regulatory framework was considered insufficient and therefore it is continuously replaced by new regulatory regimes. In banking it is the legislative amendment Basel II (currently its refinement within legislative framework Basel III[1] is being prepared). In insurance, it is the system Solvency II[2], which replaces current Solvency I regime (the directive should be fully implemented in the year 2016, see EIOPA (2013)).

Both regimes are based on the system of three pillars, which are concentrating on capital requirements for individual risks, area of risk management, controlling, rights and responsibilities of regulatory bodies and obligations pertaining to information release and institution's transparency.

Legislatively respected method for determination of capital requirements is method Value at Risk (VaR), which is widely used in the area of finance both for risk management and for regulatory purposes. No standard method of calculation for VaR exists. One of the possibilities how to determine

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[1] More detailed information about this problematic can be found at website of Bank for International Settlements
[2] More detailed information pertaining to problematic Solvency II can be found at website of European Insurance and Occupational Pensions Authority

VaR is the utilization of Monte Carlo simulation (MC). The method is based on utilizing large amount of portfolio's value development simulations. With the emphasis on improving the estimation's effectiveness, thus decreasing the estimation's error and decreasing the amount of generated scenarios, various methods of this simulation technique were introduced, in literature referred to as reduction methods, see Boyle et al. (1997).

Objective of this paper is determination of capital requirements for currency risk in insurance and bank sector by various methods of Monte Carlo Simulation. Capital requirements are expressed based on Solvency II and Basel II as Value at Risk at significance levels 85 %, 99 % and 99.5 %. For determination of Value at Risk will be applied follows methods Monte Carlo method, Antithetic Sampling Monte Carlo and Latin Hypercube Sampling with dependence Monte Carlo.

2. Value at Risk

In practice, developed and widely used method for measuring and managing risks is the value index of risk, also called Value at Risk. This method can be used to calculate capital requirements, financial risks management, integration of risks into one value etc. VaR represents the risk value, which with given probability α will not be exceeded during certain time period N, see Hull (2007). Mathematically VaR can be expressed as one sided quantile of distribution of profits and losses for certain time of holding, and it is determined based on certain historical period. It is a function which consists of two parameters: time horizon (N) and the significance level (α %). A formal equation follows:

$$\Pr\left(\Delta \tilde{\Pi} \leq -VaR\right) = \alpha. \quad (1)$$

VaR is, from theoretical point of view, relatively simple and understandable conception; however, practical determination can be a significant statistical problem. No standard calculation method exists for the determination of VaR. The differences among individual methods lie especially in the methods of simulating changes of risk factors and in methods transforming risk factors to change portfolio's value. Alexander (2008) shows, that there are three basic methods used to calculate VaR in practice: **Variance and covariance method**, which is used for estimation of potential portfolio's losses volatility and correlation, which are acquired from historical data, **Historical simulation**, where the potential future loss is estimated based on losses which happened in the past, **Monte Carlo simulation**, which works with large number of simulations of portfolio's value development and which will be further described in detail.

In case of bank subject, the capital requirement (CR) of market risk is outlined as VaR at 99% significance level for 10 days' time horizon supposing liquidation positions are held. The banks have to determine this requirement daily[3]. In case of insurance subject Solvency Capital Requirement (SCR), which corresponds to VaR, the 0.5% significance level is defined. SCR is calculated once a year.[4] Due to long-term investment horizon Minimum Capital Requirement (MCR) is further defined. It represents such a capital value under which the insurers will be subjected to unacceptable risk level. To determine MCR, the level of risk in primary capital of insurance is used at the significance level 85 % and at the time horizon of one year. At the same time, MCR must comply with other requirements based on directives Solvency II.[5]

[3] Directive of European Parliament and Council 2006/49/ES dated June 14th, 2006 about capital adequacy of investment companies and credit institutions (amended).

[4] Directive of European Parliament and Council 2009/138/ES dated November 25th, 2009 about the approach to insurance and securing activity and their performance (Solvency II).

[5] Directive of European Parliament and Council 2009/138/ES dated November 25th, 2009 about the approach to securing activity and its performance (Solvency II).

3. Monte Carlo simulation

Monte Carlo simulation (MC) is a flexible tool modeling stochastic processes and is used to determine the value of non-linear instruments or can be used where mathematical methods fail, see Alexander (2008). Method is deriving from the law of big numbers; where the large numbers of randomly generated risk factors with selected characteristics come close to theoretical assumption, see Tichý (2010). When executing Monte Carlo simulation the following procedure can be used; based on selected probability distribution (e.g. Gaussian, Poisson, Student's distribution etc.), a vector of random numbers is generated, see below. In case that the portfolio does not contain more files, it is necessary to estimate also correlation structure by e.g. Cholesky algorithm:

$$\vec{z}^{T} = \vec{\varepsilon}^{T} \cdot P, \tag{2}$$

where $\vec{\varepsilon}^{T}$ represents vector of independent, random variables from distribution $N(0;1)$, P represents upper triangular matrix derived from covariance matrix C, so called Cholesky's matrix and $\vec{z}^{T}$ is transposed vector $\vec{\varepsilon}^{T}$.

Subsequently development of yield (x) of assets with selected model specifying behavior of individual portfolio instruments is simulated e.g. Brownian motion, Levy's model etc. Specifically for Brownian motion, which will be used in the paper, the development of yields can be defined as:

$$x^{i} = \mu \cdot \Delta t + \sigma \cdot \tilde{z}^{i} \cdot \sqrt{\Delta t}, \tag{3}$$

where μ is average yield , σ is standard deviation, $\tilde{z}$ is a random number from normalized normal distribution $N(0;1)$, see (2), Δt is increase of time, *i* expresses *i*-th asset.

For generation of random numbers can be selected various probability distribution. In this case will be used to normal and student probability distribution. The normal distribution is a very commonly occurring continuous probability distribution in statistics and also in most applications in finance. Normal distribution is characterized by two parameters; mean value (μ) and standard deviation (σ). Special case of normal probability is standard normal distribution, when the mean value is equal to zero and standard deviation is equal to one. The next will be used to student distribution, which is used in finance as probabilistic models assets returns. It is symmetric and bell-shaped, like the normal distribution, but has heavier tails. This distribution is characterized by three parameters namely mean value (μ), standard deviation (σ) and degrees of freedom (υ).The student distribution becomes closer to the normal distribution when the parameter υ increase. It can be used in situation where the sample size is small and population standard deviation is unknown. Is used to many number of scenarios, thus must be worked with multivariate distribution, which not calculate with numbers, but it calculate with vectors of numbers.

3.1 Methods of Monte Carlo simulation

As it was mentioned above, MC is based on generating large numbers of random scenarios, whose selected characteristics will come close to theoretical assumption. Estimation's error than corresponds to standard deviation of result. In 1997 Boyle et al., introduced techniques, which are trying to lower estimation error (dispersion of result) and by this increasing simulation's Monte Carlo effectiveness. At the same time, it comes to reducing the number of generated scenarios and decreasing the time requirement of Monte Carlo simulation. Among these procedures of MC belong: Antithetic Sampling Monte Carlo, Stratified Sampling Monte Carlo, Control variants Monte Carlo, Moment matching Monte Carlo and others.

When applying **Monte Carlo simulation (PMC)** random elements are generated so that they correspond to characteristics of selected probability distribution. This technique is relatively quick, but the estimation will be sufficiently accurate only for large number of random scenarios. Large numbers of scenarios lead to higher time demand of simulation, see Tichý (2010). From this reason technique which enables to improve effectiveness of SMC, can be applied; e.g. Glasserman (2004) or Jäckel (2002) are dealing with this.

Antithetic Sampling Monte Carlo (ASMC) is for its simplicity and comprehensibility used very often in finance. The method is based on negative correlation among vectors of random elements, meaning $\rho(X,\overline{X})=-1$, see Tichý (2010). Supposing X random elements $\tilde{z}$ from normalized normal distribution then by multiplying vector with coefficient -1 we will get vector $\overline{X}$ of random elements $-\tilde{z}$. By unification of both vectors we can achieve double the amount of random elements, which better fulfill characteristics of selected distribution. It is a method, which leads to the decrease of the time consumption and achievement of zero mean, meaning the symmetry of probability distribution. The method's limitations lie in the fact that it can be used only when generating random elements from symmetric probability distributions.

Stratified Sampling Monte Carlo (SSMC) is another very effective method, which in comparison to ASMC is determined by more complex procedure when it comes to division of random elements sample to smaller parts in a way that probability of random elements' occurrence $\tilde{z}$ from selected probability distribution will be the same for all partial parts. The method is useful when generating numbers with almost required characteristics for low number of scenarios and can be used also in other than just symmetrical distribution. SSMC can be divided to path-integral stratification method for given distribution or non-integral method where the inverse transformation is used. Latin hypercube sampling or Bridge sampled represent more sophisticated procedures of SSMC, see Jäckel (2002), Glasserman (2004) or Tichý (2010).

Latin hypercube sampling (LHS) represents a procedure which efficiently extends stratified sampling to random vectors whose components are independent. This method is similar to stratified selection; its only difference is that for each variable the sequence of sub-intervals are permuted separately. This method enables generating two or more mutually independent groups of random numbers and can be used for generating processes, which consist of random numbers from the distributions with various characteristics. It is generated *i* independent samples (k1…kd) and is generated *m* independent permutations (P1…Pd) of {1…n} drawn from the distribution that makes all permutations equally probable. LHS is given by:

$$L=\frac{P_i^j-1}{n}+\frac{K_i^j}{n} \quad j=1,\dots d, \quad i=1,..n \tag{4}$$

This method can be with dependence (LHSD). Essentially, LHSD extends LHS to random vectors with dependent components. Proceed as above practice, but correlation matrix must be equal correlation matrix of sort generated component.

4 The application of Monte Carlo simulation

This chapter contains definitions of input data, solution and evaluation selected problem.

4.1 Problem's definition

In the case of internationally oriented market subjects the role of currency risks plays an important role; this derives from unexpected changes of foreign currency exchange rates and as a result it changes the market price and also the subject market's position. Since insurers invest especially on European and American markets, the daily closing prices of three stock indexes, which are denominated in three different currencies, were used as input data: Down Jones Industrial Average (DJI) denominated in USD, Deutscher Aktien IndeX (DAX) denominated in EUR and FTSE 100 (FTSE) denominated in GBP. Prices of individual indexes were inquired on daily basis in the period of January 1st, 2003 to January 1st, 2013 (D.).[6] For the same period exchange rates of foreign currencies to CZK were also inquired. The exchange rates announced by Czech National Bank[7] were considered. In case that some stock markets did not trade, the missing data were substituted by the value from

[6] Data available at: <http://finance.yahoo.com/>

[7] Data available at :< http://www.cnb.cz/cs/index.html>

previous business day. We have available time line of 2591 daily logarithmic yields of stock markets and exchange rates.

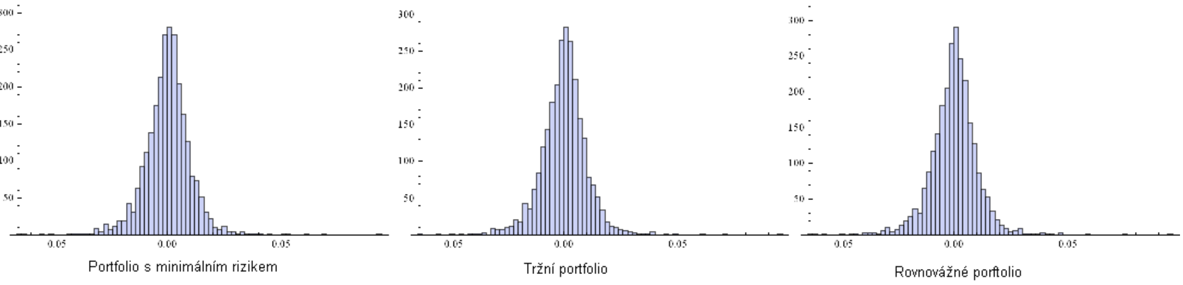
Further, we suppose three portfolios, which are: portfolio with minimal risk (A), market portfolio (M) and balanced portfolio (B). Portfolios were determined based on conditions of Markowitz model and their composition is shown in Tab. 1.

Table 1: Portfolio´s composition

	DJI (%)	**DAX (%)**	**FTSE (%)**	**E (R_p) (%)**	**σ (R_p) (%)**	**S**	**K**
A	25.6835	31.4803	42.8363	0.0066	1.0372	0.1212	9.6736
M	28.9679	32.5953	38.4367	0.0069	1.0387	0.1349	9.7632
B	33.3333	33.3333	33.3333	0.0072	1.0450	0.1450	9.8462

Source: author's calculations

Figure 1: Probability distribution for empirical dates



Source: author's calculations

From the aforementioned Table 1 or Figure 1 it is evident that the portfolios do not have normal distribution. The values of descriptive characteristics for portfolios reached very similar values. Mean value is approximately 0.007 % and standard deviation is 1.04 % for market portfolio and portfolio with minimum risk. The balanced portfolio is characterized by higher values in all descriptive characteristic. The data are more or less symmetrically distributed around mean value; there is identified low positive skewness (S) and higher kurtosis (K) compared to normal distribution. Higher kurtosis is typical for financial assets.

Table 2: Correlation matrix

	DAX	FTSE	DJI
DAX	1	0,1589	0,5121
FTSE	0,1589	1	0,3871
DJI	0,5121	0,3871	1

Source: author's calculations

The table 2 illustrated correlation matrix, which describes dependency between variables. It is symmetric, the elements on the main diagonal are equal to one and others assume values of interval (-1, 1). This values is called correlation coefficient. If the correlation coefficient is zero, the variables of the vector independence (uncorrelated) and vice versa. Very slight dependence is between indexes FTSE /DAX and FTSE/DJI. The correlation between DJI/DAX is 0.5, but it is also lower dependence. This information is used to for calculation of Cholesky´s matrix.

VaR will be calculated by methods PMC, ASMC and LHSD, at the significance levels and time horizon, which correspond to SCR, MCR a CR, meaning at 0.5% and 15% significance level for a yearlong time horizon and at 1% significance level for 10 days' time horizon. We suppose that yields have multi-dimensional normal distribution and multivariate student distribution with nine degrees of freedom, behavior of individual portfolio's instrument follows Brownian motion and 100 000 random scenarios are generated. Subsequently, used methods will be compared. Calculations were performed in Wolfram Mathematica 8.0.

Then, the data of initial period will be divided into four parts based on the development of economic situation on market, and capital requirements will be compared based on these periods. First period (I.) is historic series for period from 1. 1. 2003 to 31. 12. 2006 (before economic crises), next part (II.) is period from 1. 1. 2007 to 29. 8. 2008 (beginning of economic crises), third period (III.) includes data from 1. 9. 2008 to 31. 12. 2010 (economic crises) and last part (IV.) is period from 1. 1. 2011 to 1. 1. 2013 (retreat of economic crises).

4.2 *Problem´s solution*

By PMC, ASMC and LHSD are estimated probability distributions for each portfolio and determined descriptive characteristics, which is shown in Table 3.

Table 3: Descriptive characteristics for probability distribution

	PMC N(μ, σ)				**ASMC N(μ, σ)**				**LHSD N(μ, σ)**			
	$E(R_p)^*$	**$\sigma(R_p)^*$**	**S**	**K**	**$E(R_p)^*$**	**$\sigma(R_p)^*$**	**S**	**K**	**$E(R_p)^*$**	**$\sigma(R_p)^*$**	**S**	**K**
A	0.0100	1.0391	0.0064	3.0105	0.0066	1.0314	0.0000	3.0015	0.0100	1.0391	0.0064	3.0105
M	0.0108	1.0402	0.0073	3.0126	0.0069	1.0329	0.0000	2.9997	0.0106	1.0402	0.0073	3.0126
B	0.0111	1.0463	0.0084	3.0140	0.0072	1.0393	-0.0000	2.9971	0.0111	1.0463	0.0084	3.0140
	PMC S(μ, σ, υ)				**ASMC S(μ, σ, υ)**				**LHSD S(μ, σ, υ)**			
	$E(R_p)^*$	**$\sigma(R_p)^*$**	**S**	**K**	**$E(R_p)^*$**	**$\sigma(R_p)^*$**	**S**	**K**	**$E(R_p)^*$**	**$\sigma(R_p)^*$**	**S**	**K**
A	0.0079	1,1801	-0,032	4,5045	0.0066	1.1747	-0,0000	3.9846	0,0079	1.1807	-0,0322	4.5046
M	0.0079	1,1820	-0,031	4.5320	0.0069	1.1763	-0.0000	3.9959	0,0079	1.1820	-0.0306	4.5320
B	0.0078	1,1891	-0,029	4.5702	0.0072	1.1837	-0.0000	4.0080	0,0078	1.1891	-0.0289	4.5702

Source: author's calculations, * in %

From the above table it can be seen that individual estimations based on multi-dimensional normal distribution does not correspond with empirical values. It differs in all monitored characteristics, especially in kurtosis, which is very low. This estimation is very close to standard normal distribution, which is characterized by zero mean and standard deviation of one. Therefore student distribution is applied. Individual estimation based on multivariate student distribution more correspond with empirical values for all methods especially at mean value and kurtosis, but standard deviation increased by approximately 0.15 p.p. and skewness decrease below zero. The kurtosis can control by parameter υ (degree of freedom). In this case is used to $\upsilon = 9$, but should be selected lower value, because this change leads to increase kurtosis. For estimation it is better to use multivariate student distribution, which is able to capture higher kurtosis and heavy tails, which is characteristic for financial assets.

A comparison of different methods shows, that the best estimation was provided by ASMC method for both probability distributions. But this method can be used only for symmetric probability distribution. The results for LHSD method and PMC method are very similar. LHSD can be used to for widely application and allows reduce number of scenarios while maintaining the accuracy of the estimate. On the other hand PMC method is very simply method and provided approximately same estimation as other using sophisticate methods for selected numbers of scenarios. The most time-consuming is LHSD method, because random values are not only generated, but they are also permuted and then all acquired data must be sorted. The least time-consuming is ASMC method, but it only generates half number of scenario.

Based on selected data and aforementioned procedures the capital requirements for currency risk were calculated for various portfolios and various time periods. The results are shown in Table 3.

Table 4: Capital requirements for given portfolios and selected periods in %

Multi-dimensional normal distribution	**PMC**	**Portfolio with minimum risk**			**Market portfolio**			**Balanced portfolio**		
		SCR	MCR	CR	SCR	MCR	CR	SCR	MCR	CR
	D.	40.0102	14.5373	7.5371	39.7779	14.3846	7.5092	39.9782	14.3316	7.5536
	I.	28.0400	6.2752	6.2578	28.1567	6.3928	6.2737	28.3825	6.6196	6.2871
	II.	53.6113	31.0737	7.4115	53.0959	30.5689	7.3777	52.9817	30.1903	7.3958
	III.	53.2816	19.2511	10.2122	52.7963	18.9312	10.1774	52.8923	18.7631	10.2412
	IV.	32.2983	10.0971	6.4570	32.1558	9.9443	6.4722	32.4343	9.7842	6.5387
	ASMC	**Portfolio with minimum risk**			**Market portfolio**			**Balanced portfolio**		
		SCR	MCR	CR	SCR	MCR	CR	SCR	MCR	CR
	D.	40.4120	15.2211	7.5183	40.3284	15.1518	7.5260	40.4482	15.2142	7.5583
	I.	28.6919	7.0395	6.2584	28.9823	7.2141	6.2747	29.2202	7.4887	6.3043
	II.	54.0166	31.6601	7.3979	53.2849	31.2305	7.3789	53.2849	30.9164	7.4143
	III.	53.8025	20.1666	10.0462	53.6644	19.9652	10.0105	53.4567	19.8751	10.0833
	IV.	32.6298	10.6927	6.4392	32.5848	10.5981	6.4883	32.8656	10.5358	6.5579
	LHSD	**Portfolio with minimum risk**			**Market portfolio**			**Balanced portfolio**		
		SCR	MCR	CR	SCR	MCR	CR	SCR	MCR	CR
	D.	40.0102	14.5373	7.5371	39.7779	14.3846	7.5092	39.9782	14.3316	7.5536
	I.	28.0400	6.2752	6.2578	28.3825	6.2737	6.2578	28.3825	6.6196	6.2871
	II.	40.0102	14.5373	7.5371	39.7779	14.3846	7.5092	39.9782	14.3316	7.5536
	III.	53.2816	19.2511	10.0588	52.7963	18.9312	10.0233	52.8923	18.7631	10.0443
	IV.	32.2983	10.0971	6.4570	32.1558	9.9443	6.4722	32.4343	9.7842	6.5387
Multi-dimensional student distribution	**PMC**	**Portfolio with minimum risk**			**Market portfolio**			**Balanced portfolio**		
		SCR	MCR	CR	SCR	MCR	CR	SCR	MCR	CR
	D.	51.7927	16.1282	9.2170	51.8942	16.1232	9.2170	52.288	16.2432	9.3097
	I.	39.0249	7.7931	7.7556	39.2386	8.0412	7.7794	39.6145	8.4167	7.8003
	II.	64.1325	32.4536	8.8913	63.5776	32.1069	8.8730	63.739	31.8896	8.9523
	III.	69.0049	21.3825	12.2763	68.7242	21.2572	12.2841	69.2468	21.3345	12.3951
	IV.	42.4738	11.4842	7.9233	42.8613	11.4126	7.9705	43.2608	11.4331	8.0867
	ASMC	**Portfolio with minimum risk**			**Market portfolio**			**Balanced portfolio**		
		SCR	MCR	CR	SCR	MCR	CR	SCR	MCR	CR
	D.	51.6518	16.2957	9.2763	51.7162	16.2081	9.2755	52.0589	16.2909	9.3276
	I.	38.6077	7.9107	7.7723	38.8231	8.0600	7.8002	39.2558	8.3512	7.8235
	II.	63.9349	32.6325	8.9207	63.7273	32.2309	8.9416	63.9995	31.9232	8.9938
	III.	68.8176	21.6578	12.3491	68.8251	21.462	12.3808	69.4859	21.3533	12.4372
	IV.	42.313	11.6555	7.9629	42.6446	11.5274	8.0207	42.9913	11.4958	8.1008
	LHSD	**Portfolio with minimum risk**			**Market portfolio**			**Balanced portfolio**		
		SCR	MCR	CR	SCR	MCR	CR	SCR	MCR	CR
	D.	51.7927	16.1282	9.2170	51.8942	16.1232	9.2170	52.288	16.2432	9.3096
	I.	39.0249	7.7931	7.7556	39.2386	8.0412	7.7794	39.6145	8.4173	7.8003
	II.	64..1325	32.4536	8.8913	63.5776	32.1069	8.8730	63.739	31.8896	8.9523
	III.	69.0049	21.3825	12.2763	68.7242	21.2572	12.2841	69.2468	21.3345	12.3951
	IV.	42.4738	11.4842	7.9234	42.8613	11.4126	7.9705	43.2608	11.4331	8.0807

Source: author's calculations

It is very important to realize that with growing level of significance also grows the value of VaR and by this also capital requirements value. The same stands for time horizon, for which VaR is calculated, with growing time horizon the requirements for held capital are growing. As a result of this, SCR will always reach the highest values because it is calculated at the highest levels of significance and for a year, on the contrary CR will reach always the lowest value especially because of the time horizon for which the VaR is determined.

From the capital requirements' comparison based on individual portfolio it is evident that capital requirements reach similar values and are different in tenths in all researched periods and both probability distributions. This is given especially by portfolio's structure, which is very similar.

More distinctive differences are found within the comparisons for individual periods. Periods I. – IV. show different stages of economic development, it can be seen, that capital requirements take under consideration risk profile of given subject and if the bank or insurance company faces higher risk, it must hold more capital for undergoing risks and vice versa.

By using multi-dimensional normal distribution it can be seen, that in methods PMC and ASMC in the period of stable economic situation I. the SCR is at the 28 % level, MCR at 6-7 % and CR at 6 % for all portfolios. In the period of economic crises inception II. the requirements for held capital got higher about 25 p.p., with SCR and MCR, with CR about 1 p.p. In III. period similar results are achieved as in previous period with SCR, however in MCR the values lowered and CR in this period grew to approximately 10 %, which correspond to theoretical specification.

Estimation of capital requirements with LHSD method better correspond with theory. There is no problem with decrease capital requirements between II. and III. time period, as PMC and ASMC. Whereas, in the period of stable economic situation I. the SCR is at the 28 % level, MCR at 6 % and CR at 6 % for all portfolios. In the period of economic crises beginning II. the requirements for held capital got higher about 12 p.p. with SCR, about 8 p.p. with MCR, and with CR about 1 p.p. In III. period SCR grew again to about 53 %, MCR to about 19 % and CR to about 10 %. In last period the values lowered in SCR to about 32 %, in MCR to about 10 % and in CR to about 6 %.

From comparison of results obtained by multivariate distribution it can be seen, that results are very similar for all methods. In the first period SCR is at the approximately 39 % level, MCR and CR ranges around 7 %. In III. period SCR grow to 68 % , MCR to 21 %, which is less than in II. period. CR also increase to 12 %. In IV. period all requirements fall approximately to same values as I. period.

Using Student´s probability distribution leads to higher capital requirements, because it captures higher kurtosis, which are characterized by financial assets.

Capital requirements have a affect on cost of equity in the institution. Cost of equity for capital requirements (CE) is defined as the loss of income from investments, which are not realized from capital, which bank or insurance companies must hold for permanent fulfillment of liabilities. CE is determined by multiplying the average cost of equity in the sector and the capital requirement. CE is developed same as capital requirements, because they are determined as percent from value of capital requirements, i.e. when must be hold higher level of capital, the CE are also higher and vice versa.

5 Conclusion

Financial institutions must hold a certain amount of capital for risk coverage. Especially risk management and solvency play a key role in the fulfillment of financial institutions 'functions. Main method for estimating capital requirements is Value at Risk. Value at Risk can be determined by the Monte Carlo simulation quite well. In this method many scenarios are generated, which leads to higher time consumption of the estimation. In order to reduce generated scenarios and lower time consumption and by this improve the effectiveness of estimation, various methods of Monte Carlo simulation can be applied.

For estimation probability distribution it is better to use multivariate student distribution, which can capture higher kurtosis and heavy tails, which is characteristic for financial assets, thus it approaching empirical data. The estimation will be better, when parameter υ is reduced, this change will cause higher kurtosis.

From results of estimation probability distribution it can be stated that individual methods provide similar estimation. The best estimation in this case was provided by ASMC method, because the results are the closest to empirical probability distribution. But this method can be used only for symmetric probability. ASMC is also the least time-consuming method. The results of PMC and LHSD are approximately the same. LHSD method is sophisticated approach, which allows us to reduce number of scenarios while maintaining the accuracy of the estimate. This method is the most time-consuming. PMC is very simple method, which provide approximately same results as other methods for selected number of scenarios.

Reason for the introduction of Solvency II is that the calculation of capital requirements was captured risk profile. This fact is confirmed by the results. In times of economic crisis, when the institution is exposed higher risk, increase also capital requirements and vice versa.

From the capital requirements comparison based on individual portfolio it is evident that capital requirements reach similar values, because portfolio´s structure is very similar.

Acknowledgement

This work was supported by the SGS project of VSB-TU Ostrava under No. SP2013/3 and Czech Science Foundation through project No. 13-13142S. The support is greatly acknowledged.

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