Leverage Requirements and Systemic Risk

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July 29, 2013

Abstract

This paper demonstrates that regulators can reduce systemic risk through leverage requirements even when exact information on individual bank risk is unavailable. We consider an environment with a large number of banks that issue risky loans and are financed through a combination of costly outside capital (equity) and insured deposits (debt). In the event of failure, banks impose externalities on society in proportion to their losses that are not internalized due to limited liability. The regulator cannot precisely control individual bank behaviour through risk-weighted capital requirements as individual bank risk is private information. Nevertheless, through a leverage requirement, the regulator can actually reduce systemic risk even when it allows riskier banks to be undercapitalized.

JEL Classification Codes: G21, G32, G38  
Keywords: capital regulation, leverage caps, moral hazard, asymmetric information

*Corresponding Author. We thank Bob DeYoung, Franco Fiordelisi, Iftekhar Hasan, Kose John, Michael Koetter, Laetitia Lepetit, Amine Tarazi and participants at the LAPE Spring 2013 Workshop and IFABS 2013 for valuable comments and suggestions.
1 Introduction

Following the financial crisis of 2008-2009, policy makers have sought to tighten regulations governing the financial sector. Specifically, the new Basel III accord on banking regulation has introduced a number of new regulatory instruments to complement traditional capital regulation. One such additional instrument is a leverage ratio requirement whereby banks are required to maintain capital in excess of a certain fraction of total assets irrespective of risk, on the premise that the leverage ratio provides an additional measure of bank risk that captures indirect or unobservable exposures.¹

The key contribution of the paper is to show that a leverage requirement, while unable to adequately control individual bank behaviour, can reduce the level of systemic risk in the presence of asymmetric information on portfolio risk.² Utilizing information on capital and leverage to infer size allows the regulator to condition leverage requirements on bank size. Yet as risk is private information, leverage requirements average out capital requirements across banks with different risks. This entails that safer banks are overcapitalized while riskier banks are undercapitalized. Nevertheless, the reduction in systemic risk from overcapitalization of safe banks generally outweighs the increase in systemic risk from the undercapitalization of risky banks.

A second contribution of the paper is to show that in the presence of asymmetric information, risk-weighted capital requirements are prone to manipulation even when precise information on bank leverage is available. The reason is that leverage provides a means to infer size but not necessarily portfolio risk. In fact, as long as size and portfolio risk are not perfectly correlated, we show that banks always prefer to misreport portfolio risk to reduce capital charges.

We consider a banking sector consisting of a large number of banks, each operated by management in the interest of owners.³ All banks are financed through a combination of insured deposits (debt) and costly outside capital (equity), and issue risky loans to entrepreneurs. Banks differ both in the quality (riskiness) of their loan portfolios, and the size of the investment opportunities available to them.

The need for regulation is motivated by the existence of negative externalities banks impose on society in the case of failure. Following Giammarino et al. (1993), we take these to be proportional to bank losses that are a function of both portfolio quality and bank size. Losses can certainly be mitigated by bank capital but as banks are protected by limited liability, they do not have incentives to hold adequate capital. The regulator’s ability to control bank behaviour is constrained as banks have private information about portfolio quality and size is not directly observable. We examine outcomes when the regulator has recourse to two sets of instruments: risk-weighted capital requirements and leverage requirements.

This paper is most closely related to Blum (2008) who also studies the use of leverage requirements in the context of asymmetric information. Blum (2008) examines how leverage restrictions, together with risk-weighted capital requirements, can fully mitigate excessive risk-taking behaviour and induce banks to truthfully reveal private information on bank risk. Therefore, leverage restrictions can be seen as complimenting standard risk-weighted capital requirements.

¹See p.1 in “Revised Basel III leverage ratio framework and disclosure requirements.”
²We define systemic risk as the risk of collapse of the entire banking sector.
³We abstract from conflicts between bank managers and owners for simplicity. However, as shown by John et al. (2000), compensation for bank managers can be an effective instrument for controlling bank risk.
However, in a more general setting, we show that uniformly reducing bank risk-appetite is never compatible with addressing adverse selection and thus leverage requirements can sometimes preferred over risk-weighted capital requirements.

This paper also contributes to the broader literature on the use of leverage as an additional instrument in the prudential supervisory toolkit. Following Adrian and Shin (2010), there is increasing evidence that leverage is an important variable in understanding balance sheet adjustments for banks. Furthermore, Geanakoplos and Pedersen (2011) argue that information regarding bank leverage is also more directly observable in comparison with portfolio allocations. As a result, the literature has argued for the need to reduce excessive bank leverage. For instance, Triki (2009) shows that limiting bank leverage is necessary to prevent asset bubbles but does not examine the case with asymmetric information nor the substitutability between risk-weighted capital requirements and leverage restrictions. Vives (2011) considers both liquidity and solvency risks and shows leverage restrictions are needed to control solvency risk but does not consider asymmetric information problems nor risk-weighted capital requirements.

2 Model

We consider a static model with three actors: depositors/investors, banks, and a regulator. We describe these in more detail below.

2.1 Depositors/Investors

Depositors/investors are risk-neutral and may invest in risk-free assets or deposits issued by the bank. They are willing to lend as long as they break-even.

2.2 Banks

The banking sector consists of a large number of banks operated by owners that are risk-neutral and act as monopolists in their local loan markets. To finance operations, banks can raise debt ($D$) in the form of deposits that are protected by deposit insurance. As a result, debt is risk-free and yields a gross return equal to the risk-free rate that we normalize to 1. In addition, banks raise capital ($K$) from outside investors at a per-unit cost of $C > 1$.

Banks generate revenues by investing funds in $L$ units of risky loans. The size of investment opportunities differ across banks with $L$ drawn from the set of $\Lambda \equiv \{L_1, L_2, \ldots, L_m\}$ where $L_1 < L_2 \cdots < L_m$. The gross return on loans ($R$) for each bank is random with returns being independent across banks. Returns depend on portfolio quality $\theta$ that differs across banks with $\theta$ drawn from the set $\Theta \equiv \{\theta_1, \theta_2, \ldots, \theta_n\}$ where $\theta_1 < \theta_2 \cdots < \theta_n$. The distribution of returns for a bank with portfolio quality $\theta$ is then $F(R|\theta)$ with support over the interval $[\underline{R}, \overline{R}]$. We denote by $G(\theta, L)$ the joint distribution of risk and size in the banking sector.

We make the following assumptions about the relationship between risk and return. First, we assume that banks with higher portfolio quality have higher returns (in a fosd sense) or formally:

**Assumption 1.**

$$\frac{\partial F(R|\theta)}{\partial \theta} < 0 \text{ for all } R \in [\underline{R}, \overline{R}] \text{ and } \theta \in [\underline{\theta}, \overline{\theta}].$$  \hfill (1)
In other words, banks with higher portfolio quality are more likely to have higher returns. We then also assume that the expected return on loans is sufficiently high to warrant investment irrespective of portfolio quality:

**Assumption 2.**

\[
\int_{R}^{R_b} R \cdot dF(R|\theta) > 1 \text{ for all } \theta \in [\theta, \overline{\theta}].
\] (2)

Finally, each bank’s balance sheet matches liabilities (debt and equity) with assets (loans):

\[D + K = L.\]

Banks are protected by limited liability and go under when they can no longer afford to repay creditors. This occurs whenever \(RL < D\) or equivalently when \(R < R_b = 1 - \frac{K}{L}\). Thus, given the same size and level of capital, banks with higher portfolio quality are less likely to default because for them low returns are less likely. The probability of an individual bank failure (or default) is \(F(R_b|\theta) = F(1 - \frac{K}{L}|\theta)\) with losses equal to \((R - 1)L + K\). Therefore, bank capital reduces the probability of default and also reduces the size of bank losses.

In the absence of regulatory intervention, banks maximize expected profits subject to participation by depositors (that requires the return on deposits to be at least 1) and the balance-sheet condition. Formally, each bank solves the following problem:

\[
\max_{D, K \geq 0} \pi^p = \int_{R_b}^{R} [RL - D] dF(R|\theta) - CK \text{ s.t. } D + K = L.
\]

The lower bound \(R_b\) in the expression above ensures that the bank’s limited liability is met.

### 2.3 Regulator

We assume there is a regulator tasked with maximizing social welfare (\(W\)) defined as expected bank profits minus the expected costs of failure. The regulator values bank profits because they capture the total surplus from socially valuable intermediation activity (each bank is a monopolist in its local loan market). However, the regulator is also concerned about bank failures because they impose significant negative externalities on society.\(^4\) Following Giammarino et al. (1993), we model the social costs of failure (\(S\)) of an individual bank with portfolio quality \(\theta\) and size \(L\) as increasing in bank losses:

\[
S = \Phi \cdot \int_{R_b}^{R} [RL - D] dF(R|\theta)
\] (3)

where \(\Phi > 1\) captures all externalities.\(^5\) For example, \(\Phi\) includes the cost of raising funds from the public via distortionary taxation to fund bank bailouts but also costs stemming from contagion due to interconnectedness of the banking sector. Note that the integrand above is always negative as returns are at most \(R_b\) in the case of failure. The assumption that failure costs are increasing in bank losses is designed to capture additional externalities associated with

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\(^4\)These may take the form of a reduction in the amount of credit available to the real economy (a credit crunch) or rapid and large-scale liquidations of bank assets that drive down prices (fire-sales). For a discussion of these externalities see Hanson et al. (2011).

\(^5\)Alternatively, the social costs may be a function of the total debt or total quantity of loans issued by the bank. These specifications yield qualitatively similar results.
the failure of larger banks. Finally, we assume that the cost of externalities is sufficiently high to warrant holding capital or:

**Assumption 3.** \( \Phi > C \).

Due to limited liability, banks do not take the social costs of failure into account when choosing capital and debt. As a result, banks take too much risk relative to the socially efficient level. The presence of failure externalities thus provides a rationale for regulating the banking sector. The regulator can reduce the expected bank losses (and thereby the costs from bank failure) by requiring banks to raise additional capital. However, regulation must balance the need to reduce the expected cost of failure with cost of additional capital.

We define a feasible allocation for a bank as particular combinations of capital and debt (i.e. a pair \( \{D, K\} \)) that satisfy the balance sheet condition. We consider regulations in the form of restrictions on feasible allocations. Specifically, the regulator can impose two types of regulations: risk-weighted minimum capital requirements or non-risk weighted leverage ratio requirements.\(^6\) In the context of the above framework, a traditional risk-weighted minimum capital requirement mandates banks to hold capital in excess of a threshold level \( K(\cdot) \). This threshold is a non-decreasing function of bank risk as measured by portfolio quality \( \theta \) but may include any observable available to the regulator. A leverage ratio restriction is a lower bound on bank leverage \( L \equiv K/L \) (say \( L \)) that banks must maintain.\(^7\)

### 2.4 Timing

As for timing, the regulator first announces regulations \( \{K(\cdot), L\} \). Banks then choose capital and debt taking into account the regulations.

### 3 Moral Hazard

We first show that when banks are protected by limited liability and are not subject to any regulation, they hold insufficient capital to internalize the expected costs of failure. In the absence of regulation a bank with portfolio quality \( \theta \) and investment opportunities \( L \) chooses capital and debt to maximize expected profits:

\[
\max_{D, K \geq 0} \pi^p = \int_{R_0}^{\bar{R}} [R L - D] dF(R|\theta) - CK \text{ s.t. } D + K = L.
\]

Using the balance sheet condition we can rewrite the problem as:

\[
\max_{K \geq 0} \pi^p = \int_{1 - \frac{K}{L}}^{1} [(R - 1)L + K] dF(R|\theta) - CK \tag{4}
\]

The marginal benefit of an additional unit of capital is simply \( \int_{1 - \frac{K}{L}}^{1} dF(R|\theta) = 1 - F(1 - \frac{K}{L}|\theta) \leq 1 \) for all banks, while the marginal cost is \( C > 1 \). Hence, without regulation, banks prefer to hold zero capital and only use debt to finance operations. We summarize these results below:

\(^6\)In accordance with the new Basel III accord, we only consider leverage ratio requirements that are not risk-sensitive.

\(^7\)Leverage is defined as the ratio of capital \( K \) to total assets, \( L \). Alternative definitions of leverage such as \( L/K \) or \( D/K \) yield qualitatively similar results.
Proposition 1. Denote by \( \{ D^p(\theta, L), K^p(\theta, L) \} \) the optimal levels of capital and debt in the absence of regulation for a bank with portfolio quality \( \theta \) and size \( L \). Then, in equilibrium \( K^p(\theta, L) = 0 \) and \( D^p(\theta, L) = L - K^p(\theta, L) = L \) for all \( \theta \) and \( L \).

In contrast, the socially optimal levels of capital and debt \( \{ D^*(\theta, L), K^*(\theta, L) \} \) balance the costs of raising capital against the expected social costs of bank failure. Thus, the regulator chooses \( \{ D^*(\theta, L), K^*(\theta, L) \} \) to maximize welfare

\[
\max_{D, K \geq 0} W = \int_{R_b} [RL - D] dF(R|\theta) - CK + \Phi \int_{R} [RL - D] dF(R|\theta)
\]

s.t. \( D + K = L \).

Rewriting the objective using the balance sheet condition we obtain:

\[
W = \int_{1 - \frac{K^*(\theta, L)}{L}}^{\pi} \left[(R - 1)L + K\right] dF(R|\theta) - CK + \Phi \int_{R} \left[(R - 1)L + K\right] dF(R|\theta).
\]

Recall that profits are strictly decreasing in capital for all banks so forcing banks to hold more capital reduces profits. On the other hand, capital also reduces the expected costs of failure. Thus, the expression above demonstrates that the socially optimal level of capital for each bank trades off bank profits against the expected costs of failure. The optimal level of capital \( K^*(\theta, L) \) is characterized by the following first-order condition:

\[
\int_{1 - \frac{K^*(\theta, L)}{L}}^{\pi} dF(R|\theta) + \Phi \int_{R} \frac{1}{L} \left[(R - 1)L + K\right] dF(R|\theta) = C.
\]

The regulator increases capital requirements till the marginal social benefit of capital (a reduction in the probability of default and bank losses) equals the marginal cost of raising capital \( C \). Re-arranging we obtain:

\[
F \left(1 - \frac{K^*(\theta, L)}{L}\right) = \frac{C - 1}{\Phi - 1} \equiv \lambda.
\]

The above expression shows that for a given portfolio quality and bank size, the socially optimal level of capital is chosen to ensure that the probability of failure equals \( \lambda \). By differentiating (6) with respect to \( \theta \), we can establish that banks with lower portfolio quality (higher default risk) must hold more capital:

\[
\frac{\partial K^*}{\partial \theta} = L \cdot \frac{\partial F(R^*_\theta)}{\partial \theta} \left/ \frac{\partial F(R^*_\theta)}{\partial R} \right. < 0
\]

as the numerator is negative due to Assumption 1, and the denominator is non-negative as \( F(\cdot) \) is non-decreasing. Also, capital requirements are increasing in bank size as larger banks have higher expected losses. To see this, we differentiate (6) with respect to \( L \) to obtain

\[
\frac{\partial K^*}{\partial L} = \frac{K^*(\theta, L)}{L} > 0.
\]

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8The probability of default is fixed because we assumed a fixed marginal cost of capital as in Blum (2008).
The change in capital required as size changes is directly proportional to bank leverage. This demonstrates the importance of using information on leverage in designing risk-weighted capital regulation when size is not directly observable.

It is also important to examine how the optimal bank leverage changes in response to changes in risk and size. In fact, for a given portfolio quality, changes in size necessitate changes in capital to maintain the same leverage as before as

$$\frac{\partial (K^*/L)}{\partial L} = \frac{\partial K^*/\partial L \cdot L - K^* \cdot 1}{L^2} = \frac{\partial K^*/\partial L}{L} \cdot \frac{L}{L^2} - \frac{K^*}{L^2} = 0 \ (\text{via (8)}).$$

(9)

However, for a given size, changes in portfolio quality necessitate changes in capital that generally allow for leverage to decline if quality improves or force leverage up if quality decreases as

$$\frac{\partial (K^*/L)}{\partial \theta} = \frac{\partial K^*/\partial \theta \cdot 1}{L} < 0 \ (\text{via (7)}).$$

(10)

Moreover, via the balance sheet condition, the socially optimal level of debt is given by:

$$D^*(\theta, L) = L - K^*(\theta, L).$$

(11)

Then, from (7), we can easily conclude that riskier banks should hold less debt while larger banks hold more debt. We summarize these results below:

**Proposition 2.** The socially optimal allocation \( \{D^*(\theta, L), K^*(\theta, L)\} \) is characterized by (6) and (11). To take into account the externalities due to bank failure, banks should hold more capital and borrow less: \( K^*(\theta, L) > K^p(\theta, L)(= 0) \) and \( D^*(\theta, L) = L - K^*(\theta, L) < L = D^p(\theta, L) \). Moreover, riskier banks should hold more capital and less debt while larger banks should increase capital and debt in proportion to size.

From (6) we can also see that all banks should hold more capital and less debt if either the social costs of failure (\( \Phi \)) increase or the costs of raising capital (\( C \)) decrease.

Next, we can show that if the regulator can observe both portfolio quality and size then the socially optimal allocation can be implemented through risk-weighted capital requirements. In addition, we can also show that when bank size is not directly observable, the regulator can still implement the social optimum using a leverage requirement.

**Proposition 3.** When size is observable, the social optimum can be implemented via the following risk-weighted capital requirement \( K(\cdot) = K^*(\theta, L) \). No additional leverage requirement is necessary. When bank size is not directly observable, the regulator can still implement the social optimum using the following leverage requirement \( \mathcal{L} \geq \mathcal{L} = K^*(\theta, L_1)/L_1 \).

**Proof.** A typical bank’s problem under risk-weighted capital regulation is:

$$\max_{K \geq 0} \int_{1 - \frac{K}{L}}^{\frac{L}{R}} [(R - 1)L + K] dF(R|\theta) - CK \ \text{s.t.} \ K \geq K^*(\theta, L).$$

(12)

Since bank profits are strictly decreasing in capital (recall that the bank would prefer to hold zero capital in the absence of regulation), the minimum capital requirement will be binding in

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9We assume throughout that the regulator can perfectly observe bank capital \( K \).
equilibrium ensuring that the bank will choose $K = K^*(\theta, L)$. Then, due to the balance sheet condition, all banks choose the socially optimal level of debt.

Now, let’s consider the case when bank size is not directly observable. Recall that from (9) the optimal level of capital, while varying with size, implies constant leverage for banks with identical portfolio quality. Also, note that any leverage requirement will always bind in equilibrium for all banks as it requires a strictly positive level of capital. Then, forcing leverage to be at least $K^*(\theta, L_1)/L_1$ ensures that all banks with portfolio quality $\theta$ choose the optimal level of capital. As this holds for all $\theta$, the socially optimal allocation can be implemented. 

This result shows that when information on portfolio quality and bank leverage is available while information on total bank exposures (size) is not available, leverage requirements indirectly capture bank exposures. Moreover, they also fully capture the systemic nature of these exposures as they implement the socially optimal allocation.

4 Moral Hazard and Adverse Selection

In this section we relax the assumption that the regulator is perfectly informed about the portfolio quality of individual banks. Instead we assume that the regulator only knows the joint distribution of $\theta$ and $L$, namely $G(\theta, L)$. However, to simplify matters, we assume that the regulator can observe the $L$ for all banks.

When the regulator is imperfectly informed about portfolio quality and uses risk assessments provided by banks to decide on appropriate levels of capital, debt and loans, the socially optimal allocations are infeasible because banks always prefer to misreport. To see this, recall that bank profits are decreasing in capital for all bank types and that in the absence of regulation each would hold zero capital. Then a bank with a given portfolio quality $\theta < \theta$ prefers to misreport its portfolio quality as $\tilde{\theta} < \theta$ to reduce capital costs.

Lemma 1. When the regulator is not informed about individual bank risk, the risk-weighted capital requirement $K(\theta, L) = K^*(\theta, L)$ is prone to manipulation.

Proof. For a bank with portfolio quality $\theta$, profits from misreporting are higher than profits
from truthfully reporting:

\[
\int_{1-K^*(\tilde{\theta},L)/L}^{R} \left[ (R - 1)L + K^*(\tilde{\theta},L) \right] dF(R|\theta) - CK^*(\tilde{\theta},L)
\]

\[
- \left( \int_{1-K^*(\theta,L)/L}^{\tilde{R}} [(R - 1)L + K^*(\theta,L)] dF(R|\theta) - CK^*(\theta,L) \right)
\]

profits from misreporting

\[
\int_{1-K^*(\tilde{\theta},L)/L}^{R} \left[ (R - 1)L + K^*(\tilde{\theta},L) \right] dF(R|\theta) - CK^*(\tilde{\theta},L)
\]

profits from truthfully reporting

\[
= C \cdot \left[ K^*(\theta, L) - K^*(\tilde{\theta}, L) \right] - \int_{1-K^*(\theta,L)/L}^{1-K^*(\tilde{\theta},L)/L} \left[ (R - 1)L + K^*(\tilde{\theta},L) \right] dF(R|\theta)
\]

\[
- \int_{1-K^*(\theta,L)/L}^{\tilde{R}} \left[ K^*(\theta, L) - K^*(\tilde{\theta}, L) \right] dF(R|\theta)
\]

\[
= [C + F(1 - K^*(\theta, L)/L)] \cdot \left[ K^*(\theta, L) - K^*(\tilde{\theta}, L) \right]
\]

\[
- \int_{1-K^*(\theta,L)/L}^{1-K^*(\tilde{\theta},L)/L} \left[ (R - 1)L + K^*(\tilde{\theta},L) \right] dF(R|\theta)
\]

\[
\geq (C - 1) \cdot \left[ K^*(\theta, L) - K^*(\tilde{\theta}, L) \right] \text{ (using integration by parts for the last integral)}
\]

\[
> 0 \text{ because } K^*(\theta, L) > K^*(\tilde{\theta}, L).
\]

The key implication of the above results is that to avoid misreporting all banks must hold an identical level of capital:

**Lemma 2.** An allocation is incentive-compatible if and only if

\[ K^*(\theta, L) = K^*(L) \text{ for all } \theta \in [\tilde{\theta}, \bar{\theta}). \]

Thus, when the regulator is not informed about the portfolio quality of individual banks, the socially optimal allocation is infeasible.

The incentive-compatible socially optimal allocation then requires choosing a non-risk weighted capital requirement \( K^*(L) \) that maximizes expected social welfare:

\[
K^*(L) = \arg \max_{K} E_{\theta} \left[ \int_{1-K/L}^{\tilde{R}} [(R - 1)L + K] dF(R|\theta) - CK 
\right.
\]

\[
+ \Phi \int_{R}^{1-K/L} [(R - 1)L + K] dF(R|\theta) \right] \tag{13}
\]

\( K^*(L) \) is characterized by the following first-order condition:

\[
E_{\theta} \left[ F \left( 1 - \frac{K^*(L)}{L} \right) \right] = \frac{C - 1}{\Phi - 1} \equiv \lambda. \tag{14}
\]

Thus, in the presence of asymmetric information the socially optimal level of capital ensures that the expected probability of bank failure conditional on size equals \( \frac{C - 1}{\Phi - 1} \). In other words, the regulator chooses capital requirements to average out the probability of default across similar sized banks. Now define \( \tilde{\theta}(L) \) such that \( K^*(\tilde{\theta}(L), L) = K^*(L) \). Then, by Proposition (2), we know that \( K^*(\theta_1, L) > K^*(L) = K^*(\tilde{\theta}(L), L) > K^*(\theta_n, L) \). In other words, for a given size,
banks with low portfolio quality ($\theta < \hat{\theta}$) are undercapitalized while banks with high portfolio quality ($\theta > \hat{\theta}$) are overcapitalized.

The necessity of these capital distortions show that the regulator cannot resolve the asymmetric information problem without tolerating some level of moral hazard. This result is in sharp contrast to Blum (2008) where no such trade-off is present and a single level of capital can address both moral hazard and adverse selection. We summarize the results of this section in the following proposition:

**Proposition 4.** In the presence of asymmetric information, risk-weighted capital requirements are prone to manipulation and thus infeasible. The socially optimal incentive-compatible level of capital $K^*(L)$ is characterized by (14). It is chosen to average out the probability of default across banks for a given size. This implies that when all banks of size $L$ hold $K^*(L)$ units of capital risky banks are undercapitalized while safer banks hold excess capital. Therefore, the regulator must tolerate some degree of moral hazard in the banking system in the presence of asymmetric information.

### 4.1 Implementation

$K^*(L)$ can be implemented via a simple leverage ratio requirement. Recall, that leverage $\mathcal{L}$ is defined as $K/L$. Now, let $\underline{\mathcal{L}}$ be a lower bound on leverage that the bank must maintain where $\underline{\mathcal{L}} = K^*(L)/L$. Then, given such a requirement, the typical bank’s problem is:

$$\max_{K \geq 0} \int_{1-K/L}^{\mathcal{L}} [(R-1)L + K] dF(R|\theta) - CK \text{ s.t. } \mathcal{L} \geq \underline{\mathcal{L}}$$

(15)

The constraint $\mathcal{L} \geq \underline{\mathcal{L}}$ is equivalent to $K \geq K^*(L)$. Then as profits for all bank types are strictly decreasing in capital, in equilibrium the leverage constraint will bind, resulting in each bank of size $L$ holding $K^*(L)$ units of capital. We summarize this below:

**Proposition 5.** The risk-independent leverage ratio restriction $\underline{\mathcal{L}} \geq K^*(L)/L$ implements the information constrained-efficient outcome while additional risk-weighted capital requirements are unnecessary.

Contrary to Blum (2008), the above result suggests that leverage ratio requirements do not complement risk-based capital regulations but rather act as substitutes.

### 5 Systemic Risk

To examine systemic risk, we first need to specify the distribution $G(\theta, L)$ in more detail. Let $N$ be the total number of banks and $n_j$ be the number of banks for a given size $L_j$ so that $\sum_j n_j = N$. Also, let $\alpha_{ij}$ be the number of banks with portfolio risk $\theta_i$ and size $L_j$ where $\theta_i \in \Theta$ and $L_j \in \Lambda$. Then, to ensure that $G(\theta, L)$ is well defined we require that

$$\sum_i \alpha_{ij} = n_j \text{ for all } j.$$ 

Finally, define $\lambda_{ij} \equiv F(1 - K^*(L_j)/L_j|\theta_i)$ be the probability of default of a bank with portfolio risk $\theta_i$ and size $L_j$ under the flat leverage requirement.
Then, the systemic risk when the regulator perfectly observes bank risk is $\lambda^N$ as the regulator chooses capital requirements such that the probability of default for each bank equals $\lambda$. The systemic risk when the regulator cannot observe bank risk perfectly is $\prod_{ij} \alpha_{ij}$. We can now show the following result.

**Proposition 6.** The use of a leverage requirement generally reduces systemic risk or formally

$$\lambda^N > \prod_{ij} \alpha_{ij}$$

**Proof.** We have

$$\lambda^N = \lambda \sum_i n_i \lambda_{ij}$$

$$= \prod_j \left( \frac{\sum_i \alpha_{ij} \lambda_{ij}}{n_j} \right)^{n_j} \quad \text{(via (14))}$$

$$> \prod_j \left( \prod_i \alpha_{ij} \lambda_{ij} \right) \quad \text{via the Arithmetic-Geometric Mean Inequality}$$

$$= \prod_{ij} \alpha_{ij} \lambda_{ij}$$

Thus, the use of a leverage requirement when exact information on bank risk is unavailable can lower systemic risk. Notice that we did not rely on any particular assumptions about the distribution $G(\theta, L)$ and thus the result above is quite general.

In addition, notice that if $\theta$ and $L$ are independent then the expectation over $\theta$ in (14) is independent of $L$. Therefore, a single leverage requirement can implement the information-constrained efficient level of capital across banks of all sizes as only $K^*/L$ matters in (14). By the same reasoning, when $\theta$ and $L$ are positively correlated so that risk decreases with size, the optimal leverage requirement should also decrease with size. Similarly, a negative correlation implies the optimal leverage requirement should increase with size. Thus, a flat leverage requirement is only efficient when risk and size are independent or alternatively when leverage provides no new information about bank risk not already captured by bank size.

**6 Conclusion**

We have shown that in the presence of asymmetric information, the regulator must generally tolerate some level of inefficient risk-taking by some banks. However, the use of a leverage ratio requirement, while not always efficient, can generally reduce systemic risk as these are not prone to manipulation unlike risk-weighted capital requirements. As a result, leverage restrictions should be part of the regulator’s toolkit even though they cannot completely remove inefficient risk-taking from the banking system.
References


